Faraday rotation effect in periodic graphene structure

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We report the magneto-optical rotation effect in a periodic graphene-sheet structure. Due to the masslessness of carriers in graphene, the magnetic response is very sensitive and the magneto-optical rotation effect is therefore significant. We predict that the Verdet constant of the periodic graphene-sheet structure is roughly $10 - 100$ times that of rare-earth-doped magneto-optical glass in the infrared region.

1 INTRODUCTION

In the early 1960s, magneto-optical (MO) properties[1] of materials started to develop rapidly because of the work of Bell Laboratories. Theoretical improvement and a lot of MO material synthesis led to developing many MO devices, such as MO modulators, MO isolators, MO sensors, magnetic optical circulators and MO memory, and even magnetometry.[2] However, rotatory power is not very high in usual materials. For instance, even for typical high-Verdet materials, such as MO glasses, the Verdet constant (VC) is only on the order of $0.1 - 1\,\mathrm{min}/(\mathrm{Oe}\cdot\mathrm{cm})$.[3] Realizing a large VC is still a theoretical and experimental challenge.

Surprisingly, Ref. 4 finds that we can obtain several degrees of rotation of the polarized direction when a linear electromagnetic wave passes through a single atomic layer of carbon, graphene. Recalling that graphene is the ultimately thin material in condensed matter physics, we regard this as a significant effect. There are also other studies[5, 6, 7, 8, 9] on this topic. For instance, Ref. 5 studies the phenomenon using the equation of motion, and Ref. 6 shows that graphene exhibits extremely broadband nonreciprocal polarization rotation at subterahertz frequencies.

So far studies have focused on rotation in monolayer, bilayer or multilayer graphene. To make a practical MO device, that is, to obtain a rotation angle on the order of one radian even when the modulating magnetic field is not strong, one may resort to bulk graphite, for instance, flexible graphite.

Here we study Faraday rotation from a different aspect. We first construct a periodic structure using graphene sheets. Assuming that the periodic length is far smaller than the wavelength of incident light in the structure, the periodic graphene structure (PGS) can be considered a medium. In this paper we study Faraday rotation when an incident linearly polarized electromagnetic wave passes through the medium (or PGS).

In such a PGS a collective resonance mode, a plasmon excitation, has a role. While there are only two parameters in monolayer or multilayer graphene,[4] incident wave (angular) frequency ω and cyclotron frequency ω_c, there is a third parameter in bulk graphene, plasmon frequency ω_p, which complicates our analysis. However, we will show that one can still produce an MO device with a high VC.

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2 ANALYSIS OF FARADAY ROTATION EFFECT

In semiclassical theory the VC is[10]

$$V = \frac{e\mu}{2m_e c}\omega\frac{dn}{d\omega}, \tag{1}$$

where c, μ, e, m_e, n and ω are the velocity of light in vacuum, permeability of the medium, charge, electron mass, refractive index of the medium, and angular frequency of the incident light respectively. The equation tells us that to increase the VC, one may choose strong dispersion materials, for instance, MO glass. However, noting that $\frac{e}{m}$ (multiplied by B_0) is the cyclotron angular frequency, we have another approach to increase the VC.

As shown by Eq. (1), the strength of the MO rotation effect in materials is determined by the difference in velocity between the right circularly polarized and left circularly polarized waves. This difference reflects the asymmetry of the carrier response to an external magnetic field. In the medium, the carrier response to the external magnetic field is described by cyclotron motion, $\omega_c = \frac{eB_0}{m_{eff}}$, where B_0 and m_{eff} are the external magnetic field and carrier effective mass respectively. However, from the expression for ω_c, one can also reduce carrier effective mass to enhance the carrier response to the external magnetic field.

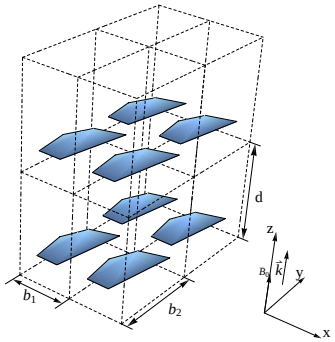

Figure 1: Periodic structure of graphene sheets. The sides of the block cell are $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{d}$, which are parallel to the x, y and z axes, respectively. We impose the modulating magnetic induction $\mathbf{B}_0$ and propagation direction of the electromagnetic wave both along z-axis.

In usual materials $m_{eff} \sim m_e$, and such enhancement is not significant. Notice the massless Dirac fermion behavior of quasielectrons in graphene,[11] *i.e.* $m_{eff} = 0$, which means the response to the external magnetic field should be sensitive. It is then natural to use graphene to get a high VC. In this section we study the Faraday MO effect in a PGS and find that we can use this PGS to get a very high VC.

We first construct the PGS as in Fig. 1. The elementary cell for such a PGS is defined as a rectangular block. The sides of the block are $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{d}$, which are parallel to the x, y and z axes, respectively, and all of them are far lower than the wavelength in the PGS. The graphene sheet outspreads in the $x-y$ plane in each elementary cell. The ratio of the area of graphene to the area of a primitive cell in the $x-y$ plane, $b_1 b_2$, is r. Here we just consider the case of $r \equiv 1$, that is, each graphene sheet completely covers the $x-y$ plane.

There are two types of fields in the structure (medium). One is the modulating static magnetic field $\mathbf{B}_0$, and the other is the electromagnetic wave field with $\mathbf{E}$ and $\mathbf{B}$. Here we assume that the modulating magnetic fields are far stronger than the electromagnetic wave fields, *i.e.* $B_0 \gg B$. We impose the modulating magnetic induction $\mathbf{B}_0$ and propagation direction of electromagnetic wave both along the z-axis. Hence $\mathbf{B}$ and electric field $\mathbf{E}$ are in the $x-y$ plane.

Landau diamagnetism is negligible because there is no external electric field. Therefore, we approximate the relative permeability as $\mu_r \approx 1$.

Now we use the concept of an effective plasmon to study dielectric permittivity. We first write the effective carrier density, $n = n_2 r/d$, where $n_2 = \frac{g}{4\pi}(\frac{E_F}{\hbar v_F})^2$ is the number of carriers per unit area, E_F is the Fermi energy (or chemical potential at zero temperature) without external fields, $v_F = 1 \times 10^6 m/s$ is the Fermi velocity of graphene and $g = 4$ is the degeneracy.[12] In the graphene sheet, the Fermi energy E_F and Fermi momentum p_F satisfy $p_F = E_F/v_F = \hbar\sqrt{\pi n_2}$.

Notice that since the Fermi velocity $v_F = \frac{c}{300} \ll c$, the long-wave limit is always correct in the periodic structure. If we turn off the modulating field, that is, we ignore the cyclotron motion, the dielectric permittivity can be written as a scalar,[13, 14, 15]

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \tag{2}$$

in the long-wave limit, where ω_p is the plasmon angular frequency or resonant angular frequency. We emphasize that, as shown in Refs. 13, 14 and 17, this form for dielectric permittivity is valid both in the quantum and classical theories, but the expression for ω_p in

the quantum theory is different from that in the classical. In the classical theory, $\omega_p = \sqrt{\frac{ne^2}{m\epsilon_0}}$, while in graphene,[13, 14, 15] $\omega_p = \sqrt{\frac{ne^2 v_F}{p_F \epsilon_0}}$ due to quantum and 'relativistic' effects, where ϵ_0 is the vacuum permittivity. In this case, the permittivity is isotropic the in $x-y$ plane and there is no rotation effect.

Now turn on B_0 along the $z-$axis and discuss the rotation effect. To make semiclassical theory apply, we first assume $\omega_c = \frac{eB_0 v_F}{p_F}, \omega_p, \omega \ll \frac{E_F}{\hbar}$.[16] Since $\hbar\omega \ll E_F$, the photon energy is much lower than E_F, so we consider only the cyclotron motion near the Fermi energy. This is because the carriers with energy far lower than the Fermi surface cannot absorb energy from a photon because of the Pauli exclusion principle. Under the assumption, as pointed out by the Appleton-Hartree theory, we write the relative dielectric permittivity ϵ_r in the $x-y$ plane as[17]

$$\epsilon_r = \begin{pmatrix} \epsilon_1 & \mathrm{i}\epsilon_2 \\ -\mathrm{i}\epsilon_2 & \epsilon_1 \end{pmatrix}, \tag{3}$$

where $\epsilon_1 = 1 + \frac{\omega_p^2}{\omega_c^2 - \omega^2}$, $\epsilon_2 = \frac{\omega_p^2 \omega_c}{\omega(\omega_c^2 - \omega^2)}$, $\omega_p = \sqrt{\frac{ne^2 v_F}{p_F \epsilon_0}}$ and $\omega_c = \frac{eB_0}{m_c} = \frac{eB_0 v_F}{p_F}$. We have shown in Ref. 17 that ω_p and ω_c are quantum angular frequencies of the plasmon and cyclotron respectively. Equation (3) tells us that the effective permittivity is determined by two physical behaviors in the PGS, namely, cyclotron motion and plasmon excitation, characterized by two frequencies, ω_c and ω_p respectively. The cyclotron motion is determined by the modulating magnetic field and the plasmon is a collective mode. On one hand we find that, if $\omega_c \ll \omega, \omega_p$, the dielectric permittivity tensor will degenerate into Eq. (2), while on the other hand, $\epsilon_2 \propto \omega_c/\omega$ will lead to the optical rotation effect.

Maxwell's equations in the PGS are

$$\begin{cases} \nabla \times \mathbf{E} &= -\frac{\partial}{\partial t}\mathbf{B}, \\ \nabla \times \mathbf{H} &= \frac{\partial}{\partial t}\mathbf{D}, \\ \nabla \cdot \mathbf{B} &= 0, \\ \nabla \cdot \mathbf{E} &= 0, \end{cases} \tag{4}$$

and constitutive relations are

$$\begin{cases} \mathbf{B} &= \mu_0 \mathbf{H}, \\ \mathbf{D} &= \epsilon_0 \epsilon_r \cdot \mathbf{E}, \end{cases} \tag{5}$$

where μ_0 is the vacuum magnetic permeability.

Supposing the electric field and magnetic field can be written as $\mathbf{B}(\mathbf{E}) = \mathbf{B}_0(\mathbf{E}_0)e^{\mathrm{i}\omega t - \mathrm{i}kz}$, where $\mathbf{B}$, $\mathbf{E}$, $\mathbf{B}_0$ and $\mathbf{E}_0$ are vectors in the $x-y$ plane, one finds that for the electric field

$$\frac{k^2 c^2}{\omega^2}\mathbf{E}_0 - \epsilon_r \mathbf{E}_0 = 0, \tag{6}$$

where the identity $\mu_0\epsilon_0 = 1/c^2$ is used. The relation for the magnetic field is similar. For each ω, corresponding to solutions of nonzero $\mathbf{E} = (E_x, E_y)$ in the above equation, there are two eigenmodes: $k_1 = \frac{\omega}{c}\sqrt{\epsilon_1 + \epsilon_2}$ with eigenvector $\mathbf{E}_{01} = (\hat{e}_x - \mathrm{i}\hat{e}_y)A/2$ and $k_2 = \frac{\omega}{c}\sqrt{\epsilon_1 - \epsilon_2}$ with eigenvector $\mathbf{E}_{02} = (\hat{e}_x + \mathrm{i}\hat{e}_y)A/2$, where A is a constant. The former eigenmode is left circularly polarized while the latter eigenmode is right circularly polarized. Since $k_1 \neq k_2$ for a certain ω, the velocities of these two circularly polarized waves are different. Therefore, a rotation effect occurs for a linear electromagnetic wave passing through the PGS; that is, if the incident wave is linearly polarized from the negative z-axis, the emitting wave is a still linearly polarized but with a rotational polarized direction.

To analyze the rotation effect, we first suppose that both left and right circularly polarized waves can propagate in the medium and write out the electric fields of these waves as

$$\begin{cases} \mathbf{E}_1 = \frac{A}{2}(\cos(\omega t - k_1 z)\hat{e}_x + \sin(\omega t - k_1 z)\hat{e}_y), \\ \mathbf{E}_2 = \frac{A}{2}(\cos(\omega t - k_2 z)\hat{e}_x - \sin(\omega t - k_2 z)\hat{e}_y), \end{cases} \tag{7}$$

with total $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$. This expression means that in the $z = 0$ plane the linear polarization is along the $x-$axis. We get

$$\begin{aligned} \mathbf{E} &= A\cos(\omega t - \frac{k_1 + k_2}{2}z)\cos(\frac{k_1 - k_2}{2}z)\hat{e}_x \\ &\quad - A\cos(\omega t - \frac{k_1 + k_2}{2}z)\sin(\frac{k_1 - k_2}{2}z)\hat{e}_y \end{aligned} \tag{8}$$

Therefore, the total wave vector is

$$k = \frac{\omega}{2c}(\sqrt{\epsilon_1 + \epsilon_2} + \sqrt{\epsilon_1 - \epsilon_2}), \tag{9}$$

and the rotatory power is

$$\alpha = -\frac{\omega}{2c}(\sqrt{\epsilon_1 - \epsilon_2} - \sqrt{\epsilon_1 + \epsilon_2}). \tag{10}$$

To consider rotatory power in detail we introduce two dimensionless quantities, $Q = \omega_c/\omega_p$ and $\omega_0 = \omega/\omega_p$. Then, $\epsilon_1 = 1 + \frac{1}{Q^2 - \omega_0^2}$ and $\epsilon_2 = \frac{Q}{\omega_0(Q^2 - \omega_0^2)}$. We find that when $\omega_0 \geq \frac{1}{2}(Q + \sqrt{Q^2 + 4})$, both $\epsilon_1 \pm \epsilon_2 > 0$, and a rotation effect occurs. Notice that if we take $k < 0$, the rotatory power is still α. This phenomenon is the well-known non-reciprocity of the MO effect.

Now, we have

$$\begin{aligned} \frac{c\alpha}{\omega_p} &= \frac{\omega_0}{2}\{\sqrt{1 + \frac{1}{Q^2 - \omega_0^2} + \frac{Q}{\omega_0(Q^2 - \omega_0^2)}} \\ &\quad - \sqrt{1 + \frac{1}{Q^2 - \omega_0^2} - \frac{Q}{\omega_0(Q^2 - \omega_0^2)}}\ \}. \end{aligned} \tag{11}$$

In fig. 2(a) we show $\frac{c\alpha}{\omega_p}$ vs. ω_0 with different Q. We find that for fixed (high) ω_0 and ω_p, $|\alpha|$ increases with decreasing ω_0 or increasing Q (and thus B_0). The behavior agrees with that of multilayer graphene, as Fig. 3 in Ref. 4 shows. (The relative negative sign is due to the fact that we take $\epsilon_F > 0$ while in Ref. 4 $\epsilon_F < 0$). At $Q = \frac{\omega_0^2-1}{\omega_0}$, α reaches its maximum value, $\frac{c\alpha}{\omega_p} = \omega_0\sqrt{\frac{\omega_0^2-1}{4\omega_0^2-2}}$, which can also be seen in Fig. 3. However, unlike single- and multilayer graphene, one cannot get a further step at this time. If we further reduce the incident frequency, one of the circularly polarized waves should decay in the medium, and the emitting wave should generally be elliptically polarized if the thickness of the medium is finite. Notice that for single and multilayer graphene, the decay is not important and we can also get a linearly polarized emitting wave.[4]

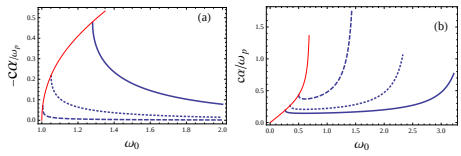

Figure 2: $\frac{c\alpha}{\omega_p}$ vs. ω_0 with different Q. (a) small Q and higher ω_0. (b) larger Q and lower ω_0. In figure (a) the dashed line, dotted line and solid line are for $Q = 0.01, 0.1,$ and 0.5 respectively and the red line is $\omega_0\sqrt{\frac{\omega_0^2-1}{4\omega_0^2-2}}$. In figure (b) the dashed line, dotted line and solid line are for $Q = 1.5, 2.5,$ and 3.5 respectively and the red line is $\omega_0\sqrt{\frac{1-\omega_0^2}{2-4\omega_0^2}}$.

Ordinarily, Q is a small quantity. Theoretically, one may also study the case of large Q. One finds that if $Q > 1/\sqrt{2}$ and $\frac{1}{2}(\sqrt{4+Q^2}-Q) < \omega_0 < Q$, both right and left circularly polarized wave can pass the PGS without decay. We show the result of lower frequency with larger Q at Fig. 2(b). Notice in this region rotatory power $\alpha > 0$. However, in this region the lower ω_p and ω_c make the quasielectron behavior dominated by quantization, thereby making our results doubtful.

Figure 2(a) shows that $|\alpha|$ is on the order of ω_p/c when the frequency of the incident wave is close to the resonant frequency. For instance, when we adjust the parameters to $E_F = 0.3\,eV$, $d = 205\,\text{nm}$ and $Q = 0.01$ ($B_0 \simeq 4.7\text{T}$), we find that $\hbar\omega_p = 0.09\text{eV}$ and $|\alpha|$ is on the order of $10 - 10^2\,\text{rad/mm}$, which is significant.

Fig. 3 shows that at small Q, rotatory power is proportional to Q, or in other words, $\alpha \propto B_0$. From the Taylor expansion we get the VC with

$$V = \frac{|\alpha|}{B_0/\mu_0} = \frac{ev_F^2\mu_0}{2cE_F\omega_0\sqrt{\omega_0^2-1}} = \frac{5.73}{x(\text{eV})\omega_0\sqrt{\omega_0^2-1}}(\text{min/Oe}\cdot\text{ cm}), \quad (12)$$

where we assume $E_F = x(\text{eV})$. In the structure we generally choose $x \sim 0.1\text{eV}$. When $Q << 1$, one way to improve V is setting The parameter to make frequency close to the resonant frequency, $\omega_0 \gtrsim 1$. If we set $\omega_0\sqrt{\omega_0^2-1} \sim 1$ and set $E_F = 0.3eV$, V is on the order of $19\,\text{min/Oe}\cdot\text{cm}$. The VC of the graphene structure is about one or two orders larger than that of MO glass. This is thus a significant advancement in magnetic optics.

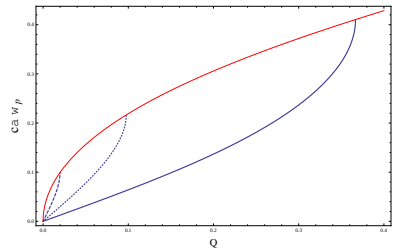

Figure 3: $\frac{c\alpha}{\omega_p}$ vs. Q with different ω_0. The dashed line, dotted line and solid line are for $\omega_0 = 1.01, 1.05, and 1.2$ respectively. The red line is $\frac{1}{2}(Q+\sqrt{4+Q^2})\sqrt{\frac{Q}{3Q+\sqrt{4+Q^2}}}$.

Equation (1) shows that V in a usual material is inversely proportional to m_{eff}, where m_{eff} is the effective carrier mass. In graphene, $m_{eff} = 0$ and the usual expression is not valid. However, p_F/v_F plays the same role as m_{eff}, and thus one can obtain a very large (not infinite) VC. Here we estimate the order of enhancement. In usual materials, the carrier mass $m_{eff} \sim m_e \simeq 10^5\,eV/c^2$. However, in graphene, cyclotron motion mass plays the role of effective mass, $m_c = E_F/v_F^2$. If we choose $E_F = 0.2eV$, we find that $m_c \sim 10^4 eV/c^2 \sim 10^{-1} - 10^{-2}m_e$. Thus, the VC in the PGS is about one or two orders larger than in rare-earth-doped MO glass.

Furthermore, as shown in Fig. 3, when Q tends to $\frac{\omega_0^2-1}{\omega_0}$, the rotatory power exhibits nonlinear behavior and VC increases further.

3 DISCUSSIONS

In this manuscript we have discussed the MO rotation effect in a PGS. The masslessness of graphene carri-

ers gives them a very sensitive response to an external field, and the VC of the PGS is very high. We predict that the VC in the structure is about one or two orders larger than in rare-earth-doped magneto-optical glass.

Furthermore, even if ω is not close to the resonant angular frequency ω_p, we may also improve B to increase the VC, although such improvement is constrained by the condition $Q < \frac{\omega_0^2-1}{\omega_0}$.

In a usual material, the effective mass is always a constant, whereas in graphene, the role of m_{eff}, $m_c = E_F/v_F^2$, is adjustable. This behavior can further expand the application range of the magneto-optical effect.

We conclude that besides MO glass, we can also use a graphene-sheet periodic structure to attain a high VC, increasing the VC up to one or two orders of magnitude. We hope our study is helpful in designing new types of MO devices.

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[16] Here we discuss briefly the constraint $\omega_c, \omega_p, \omega \ll E_F/\hbar$. Supposing $E_F = x\,eV$, $\omega_c \ll E_F/\hbar$ means that $B_0 \ll \frac{E_F^2}{e\hbar v_F^2} \sim x^2 \times 10^3 T$, which is a very weak constraint. At the same time, $\omega_p \ll E_F/\hbar$ means that $d \gg 5.75/x\,nm$, which is still not a strong constraint. But because of the above constraint for d, one cannot use common bulk graphite. To satisfy the condition we may resort to flexible graphite.[18] Finally, $\omega \ll E_F/\hbar$ means that $\omega \ll 1.5x \times 10^{15} s^{-1}$. Correspondingly, if we choose $x = 0.2$, then the constraints are $B_0 \ll 40\,T$, $d \gg 29\,nm$ and $\omega \ll 3 \times 10^{14}\,s^{-1}$ respectively.

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